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Measurement of SEA coupling loss factors using point mobilities

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In this paper calculation of SEA coupling loss factors, using measured point mobility, is derived for coupled systems, homogeneous or not, with rigid or soft links. Some simplifications and hypothesis are necessary to fit with SEA basic relations. To validate the theory an experiment was done on plates coupled in three points; the agreement is reasonably good for homogeneous and non-homogeneous plates.

1. Introduction

The SEA method (see Lyon & Maidanik 1962; Scharton & Lyon 1968) offer a good tool to analyse and predict acoustic and vibration transmissions in coupled systems (see Craik (1982) for building structures and Plunt (1980) for ships). From the practical point of view, the capital element for applying SEA is the determination of the coupling loss factors (CLF). Theoretical estimations of CLF can be obtained in simple cases from wave propagation in infinite coupled beams, plates, shells (for plates see Gibbs & Guilford 1976, 1987; Wohle *et al.* 1981; Van Backel & De Vries 1983). In general cases Keane & Price (1987) gave a relation for CLF using direct and cross receptance of coupled systems. It is also possible to derive the CLF from Green's function of decoupled structure, as demonstrated by Davies & Wahab (1981) on two coupled beams.

When people deal with real industrial structures, difficulties arise; the structures are non-academic due to complicated shapes and heterogeneities, the links are often non-rigid, dissipative and localized at some points. In this case it is impossible to calculate theoretically the coupling loss factors, except if a finite element modelization is used (see Simmons (1991) for example). However, this technique is not easy to use in the acoustic frequency range. A second possibility consists of measuring the CLF, by an experimental identification (see Lalor 1987). It is based on an inverse SEA procedure, which calculates the CLF, from the energies of the coupled systems. Two difficulties can be noticed; the systems must be coupled to apply the method, thus it is impossible to predict CLF from measurement on decoupled structures. The calculation of CLF from energies necessitates the resolution of a linear system that can be ill conditioned, this is related to the relative insensibility of SEA energy prediction, to modifications of coupling loss factors.

The study of structures coupled by point links, can be made with the mobility technique; one has to characterize the coupling points with their direct and transfer mobilities as presented, for example, by Hemingway (1986). On complicated structures (e.g. industrial) the mobilities must be measured. These measurements, contrary to inverse SEA procedure, are done on decoupled structures. The difficulties in applying this method are because of the appearance of singular frequencies, due to the cumulation of errors, in particular when the number of links is large. The precise measurement of mobilities is thus very important for this technique, unfortunately the uncertainty is large, especially on the phase and when frequency is high. It seems interesting to us to mix the two approaches to calculate CLF with measured mobilities. This is interesting from two points of view: (i) the CLF are determined from measurements on decoupled structures; and (ii) only the active power is necessary. The reactive part, difficult to measure, can be ignored.

2. Direct and transfer mobilities

Let us consider two systems coupled in some points, and excited at a given angular frequency ω. The problem consists in calculating the coupled response, using decoupled behaviour, of each system. This can be done by introducing mobilities $M_{\mathrm{I}}(Q_i, Q_j)$ defined as the velocity of system I at point Q_i when point Q_j is excited by a force of unit amplitude and angular frequency ω. It is a complex quantity depending on frequency. If the two systems are excited they have a velocity before coupling of $\tilde{V}_{\mathrm{I}}(Q_i)$ and $\tilde{V}_{\mathrm{II}}(Q'_j)$, the velocity of each structure after coupling can be obtained at each frequency with relations (1) and (2), based on linearity of the systems and links:

$$V_{\mathrm{I}}(Q_i) = \tilde{V}_{\mathrm{I}}(Q_i) + \sum_{j=1}^{N} M_{\mathrm{I}}(Q_i, \bar{Q}_j)\, F^{\mathrm{c}}_{\mathrm{I}}(\bar{Q}_j), \tag{1}$$

$$V_{\mathrm{II}}(Q'_i) = \tilde{V}_{\mathrm{II}}(Q'_i) + \sum_{j=1}^{N} M_{\mathrm{II}}(Q'_i, \bar{Q}'_j)\, F^{\mathrm{c}}_{\mathrm{II}}(\bar{Q}'_j), \tag{2}$$

where an overbar indicates a coupling point, and a prime a point of system II. $\bar{Q}_i$ and $\bar{Q}'_j$ are the coupling points respectively on system I and II. The coupling forces acting on system I and II are $F^{\mathrm{c}}_{\mathrm{I}}(\bar{Q}_j)$, $F^{\mathrm{c}}_{\mathrm{II}}(\bar{Q}'_j)$. To calculate the coupling forces, in the case of rigid links, one has to write equality of velocity and equilibrium of forces, at the coupling points:

$$V_{\mathrm{I}}(Q_i) = V_{\mathrm{II}}(Q'_i), \tag{3}$$

$$F^{\mathrm{c}}_{\mathrm{I}}(\bar{Q}_j) + F^{\mathrm{c}}_{\mathrm{II}}(\bar{Q}'_j) = 0. \tag{4}$$

After calculation one obtains

$$\{F^{\mathrm{c}}_{\mathrm{I}}(\bar{Q}_i)\} = [M_{\mathrm{I}}(\bar{Q}_i, \bar{Q}_j) + M_{\mathrm{II}}(\bar{Q}'_i, \bar{Q}'_j)]^{-1}\{\tilde{V}_{\mathrm{II}}(\bar{Q}'_j) - \tilde{V}_{\mathrm{I}}(\bar{Q}_j)\}. \tag{5}$$

3. The case of one rigid link

To simplify the analysis, let us consider only one link, equation (5) reduces to

$$F^{\mathrm{c}}_{\mathrm{I}}(\bar{Q}_1) = \frac{\tilde{V}_{\mathrm{II}}(\bar{Q}'_1) - \tilde{V}_{\mathrm{I}}(\bar{Q}_1)}{M_{\mathrm{I}}(\bar{Q}_1, \bar{Q}_1) + M_{\mathrm{II}}(\bar{Q}'_1, \bar{Q}'_1)}. \tag{6}$$

It is now easy to calculate the power injected in system I and II:

$$\Pi^{\mathrm{I}}(\bar{Q}_1) = \tfrac{1}{2}\mathrm{Re}\{F^{\mathrm{c}}_{\mathrm{I}}(\bar{Q}_1)\, V^{*}_{\mathrm{I}}(\bar{Q}_1)\}, \tag{7}$$

$$\Pi^{\mathrm{II}}(\bar{Q}'_1) = \tfrac{1}{2}\mathrm{Re}\{F^{\mathrm{c}}_{\mathrm{II}}(\bar{Q}'_1)\, V^{*}_{\mathrm{II}}(\bar{Q}'_1)\}. \tag{8}$$

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The rigid link being non dissipative, the injected power in each systems are opposite. The power going out of system II through the link is equal to the injected power in system I:

$$\Pi_{\mathrm{II}\to\mathrm{I}} = \tfrac{1}{4}\mathrm{Re}\{(\tilde{V}_I + \tilde{V}_{\mathrm{II}})F^* + (M_{\mathrm{I}} - M_{\mathrm{II}})FF^*\}. \tag{9}$$

To obtain (9) one has to use (7) and (8) and simplify the notation

$$\tilde{V}_{\mathrm{I}} = \tilde{V}_{\mathrm{I}}(\bar{Q}_1), \quad M_{\mathrm{I}} = M_{\mathrm{I}}(\bar{Q}_1, \bar{Q}_1), \quad \tilde{V}_{\mathrm{II}} = \tilde{V}_{\mathrm{II}}(\bar{Q}'_1), \quad M_{\mathrm{II}} = M_{\mathrm{II}}(\bar{Q}'_1, \bar{Q}'_1),$$
$$F^{\mathrm{c}}_{\mathrm{I}}(\bar{Q}_1) = -F^{\mathrm{c}}_{\mathrm{II}}(\bar{Q}'_1) = F.$$

Replacing F with expression (6) gives, after calculations,

$$\Pi_{\mathrm{II}\to\mathrm{I}} = \frac{1}{2}\left[|\tilde{V}_{\mathrm{II}}|^2 \frac{\mathrm{Re}\,(M_{\mathrm{I}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2} - |\tilde{V}_{\mathrm{I}}|^2 \frac{\mathrm{Re}\,(M_{\mathrm{II}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right] + \frac{1}{2}\left[\frac{\mathrm{Re}\,(\tilde{V}_{\mathrm{I}}\,\tilde{V}^*_{\mathrm{II}} M_{\mathrm{II}} - \tilde{V}^*_{\mathrm{I}}\,\tilde{V}_{\mathrm{II}} M_{\mathrm{I}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right]. \tag{10}$$

This equation is valid for a single frequency analysis, in the case of band excitation it must be integrated over frequency. When non-correlated vibrations are considered the third term of (10) integrated over frequency is neglected. Equation (10) reduces to

$$\langle \Pi_{\mathrm{II}\to\mathrm{I}} \rangle = \frac{1}{2}\left\langle |\tilde{V}_{\mathrm{II}}|^2 \frac{\mathrm{Re}\,(M_{\mathrm{I}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right\rangle - \frac{1}{2}\left\langle |\tilde{V}_{\mathrm{I}}|^2 \frac{\mathrm{Re}\,(M_{\mathrm{II}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right\rangle, \tag{11}$$

where brackets signify integration over a frequency band $\Delta\omega$.

If in addition one supposes that decoupled vibration fields vary smoothly with frequency, it is possible to approximate (11) with

$$\langle \Pi_{\mathrm{II}\to\mathrm{I}} \rangle = \frac{1}{2}\frac{\langle|\tilde{V}_{\mathrm{II}}|^2\rangle}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{I}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right\rangle - \frac{1}{2}\frac{\langle|\tilde{V}_{\mathrm{I}}|^2\rangle}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{II}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right\rangle. \tag{12}$$

Physically this assumption corresponds to systems having sufficiently high modal overlap. This equation relates the power flow from system II to system I, with local velocities of both systems before coupling.

To introduce energies one has to assume that the vibration fields are homogeneous and so vary slowly with location. In homogeneous systems it was demonstrated by Dowell & Kubota (1985), except near the boundary. The equality of kinetic and deformation energies is assumed, thus for decoupled system, the total energy over the bandwidth $\Delta\omega$ is

$$\langle \tilde{E}_{\mathrm{I}} \rangle = \tfrac{1}{2}\langle[\langle\langle \mu_{\mathrm{I}} |\tilde{V}_{\mathrm{I}}|^2 \rangle\rangle]\rangle \approx \tfrac{1}{2}\langle\langle \mu_{\mathrm{I}} \rangle\rangle\, \langle |\tilde{V}_{\mathrm{I}}|^2 \rangle, \tag{13}$$

where the density of mass of system I is μ_{I} and the double bracket represents spacial integration over the system. The following can be applied if the coupling point is not situated close to boundary. By using (12) in connection with (13) gives

$$\langle \Pi_{\mathrm{II}\to\mathrm{I}} \rangle = [\eta_{\mathrm{II\,I}}\langle \tilde{E}_{\mathrm{II}} \rangle - \eta_{\mathrm{I\,II}}\langle \tilde{E}_{\mathrm{I}} \rangle]\,\omega, \tag{14}$$

with

$$\eta_{\mathrm{II\,I}} = \frac{1}{m_{\mathrm{II}}\,\omega}\left[\frac{1}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{I}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right\rangle\right], \tag{15}$$

$$\eta_{\mathrm{I\,II}} = \frac{1}{m_{\mathrm{I}}\,\omega}\left[\frac{1}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{II}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right\rangle\right], \tag{16}$$

where $m_{\mathrm{I}} = \langle\langle \mu_{\mathrm{I}} \rangle\rangle$ is the total mass of the system and ω is the centre band frequency.

A last assumption is necessary to identify with SEA relation, it is the weak coupling hypothesis in the sense of energies varying weakly before and after coupling. Let us introduce the energies after coupling E_{I} and E_{II}:

$$\langle E_{\mathrm{I}}\rangle = \langle \tilde{E}_{\mathrm{I}}\rangle + \langle e_{\mathrm{I}}\rangle \quad \text{and} \quad \langle E_{\mathrm{II}}\rangle = \langle \tilde{E}_{\mathrm{II}}\rangle + \langle e_{\mathrm{II}}\rangle. \tag{17}$$

Equation (14) gives

$$\langle \Pi_{\mathrm{II}\to\mathrm{I}}\rangle = \omega[\eta_{\mathrm{II\,I}}\langle E_{\mathrm{II}}\rangle - \eta_{\mathrm{I\,II}}\langle E_{\mathrm{I}}\rangle - \eta_{\mathrm{II\,I}}\langle e_{\mathrm{II}}\rangle + \eta_{\mathrm{I\,II}}\langle e_{\mathrm{I}}\rangle], \tag{18}$$

and the weak coupling assumes that one can neglect the two last terms in (18).

$$\left.\begin{aligned} \langle \Pi_{\mathrm{II}\to\mathrm{I}}\rangle &\approx \omega[\eta_{\mathrm{II\,I}}\langle E_{\mathrm{II}}\rangle - \eta_{\mathrm{I\,II}}\langle E_{\mathrm{I}}\rangle], \\ \eta_{\mathrm{II\,I}} &= \frac{1}{m_{\mathrm{II}\omega}}\left[\frac{1}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{I}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right\rangle\right], \\ \eta_{\mathrm{I\,II}} &= \frac{1}{m_{\mathrm{I}\omega}}\left[\frac{1}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{II}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right\rangle\right]. \end{aligned}\right\} \tag{19}$$

This equation is the basic SEA relation, it shows that coupling loss factors can be calculated from point mobilities with (19).

4. The case of several rigid links

Equation (19) is established for one rigid link. In a practical situation several links must often be considered and an extension to this case is of interest. When several links are coupling the two systems, the exact calculation of powers exchanged is cumbersome, as transfer mobilities must be used. From the experimental point of view, measurement of direct and transfer mobilities is not easy when the number of links increases. So an important question arises: are transfer mobilities strongly modifying the transmitted power from one system to the other? In other words is each link exchanging power independently from the others? To answer this question we consider a system of two plates coupled by three rigid links. The geometry of the system is the same as the one used in experiment (see figure 2) but plates here are simply supported to simplify the calculation. The kinetic energy T of the receiver plate II is calculated when a unit force is applied at point E on the plate I. Figure 1 presents the difference in dB between the exact solution T_{e} and approximation neglecting transfer mobilities T_{a}. For systems of high modal density and when an average over frequency is done, the influence of transfer mobility is negligible.

In consequence coupling loss factors for several rigid links can be obtained by summation of expressions (19) over points of coupling

$$\eta_{\mathrm{II\,I}} = \frac{1}{m_{\mathrm{II}}\,\omega}\sum_{k=1}^{N}\frac{1}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{I}}^k)}{|M_{\mathrm{I}}^k+M_{\mathrm{II}}^k|^2}\right\rangle, \tag{20}$$

$$\eta_{\mathrm{I\,II}} = \frac{1}{m_{\mathrm{I}}\,\omega}\sum_{k=1}^{N}\frac{1}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{II}}^k)}{|M_{\mathrm{I}}^k+M_{\mathrm{II}}^k|^2}\right\rangle. \tag{21}$$

The exponent k indicates the link number, and N is the total number of coupling points. For large values of N, where the coupling points become close to each other

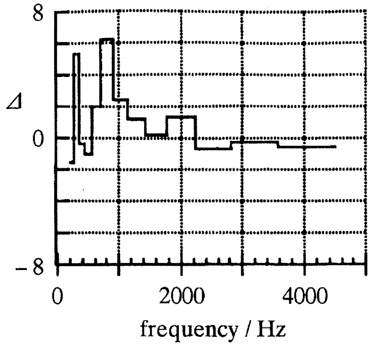

Figure 1. Influence of the transfer mobility: $\Delta = 10\log(T_e) - 10\log(T_a)$ against frequency in Hertz.

it is important to verify that the transfer mobilities can be still neglected. In other words links must be sufficiently distant otherwise it is necessary to use a link or surface effective mobility (see, for example, Naji *et al.* 1992; Petersson & Plunt 1982).

5. The case of non-homogeneous structures

In the previous analysis, vibration fields are supposed homogeneous, for industrial structures this assumption is not true in general. If, as previously assumed, decoupled vibrations are non-correlated, and structures are coupled rigidly in one point, equation (12) remains valid. The difference between homogeneous and non-homogeneous systems comes from equation (13) as local quadratic velocity can vary strongly in non-homogeneous systems. Let us introduce the homogeneity factor g calculated on the decoupled structure with their own excitation, over the bandwidth frequency $\Delta\omega$. It is the non-dimensional ratio indicated by (22). Physically it corresponds to the ratio of coupling point energy with the total mass of the system over total energy of the system.

$$g_{\mathrm{I}} = \frac{\langle |\tilde{V}_{\mathrm{I}}(\bar{Q}_1)|^2 \rangle m_{\mathrm{I}}}{2\langle \tilde{E}_{\mathrm{I}} \rangle} \quad \text{and} \quad g_{\mathrm{II}} = \frac{\langle |\tilde{V}_{\mathrm{II}}(\bar{Q}'_1)|^2 \rangle m_{\mathrm{II}}}{2\langle \tilde{E}_{\mathrm{II}} \rangle}. \tag{22}$$

This homogeneity factor is equal to 1 for homogeneous vibration field. The equation (14) can then be used but with modified coupling loss factors

$$\left.\begin{aligned} \eta_{\mathrm{II\,I}} &= \frac{g_{\mathrm{II}}}{m_{\mathrm{II}}\,\omega}\left[\frac{1}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{I}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right\rangle\right], \\ \eta_{\mathrm{I\,II}} &= \frac{g_{\mathrm{I}}}{m_{\mathrm{I}}\,\omega}\left[\frac{1}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{II}})}{|M_{\mathrm{I}}+M_{\mathrm{II}}|^2}\right\rangle\right]. \end{aligned}\right\} \tag{23}$$

For several coupling points which verify the assumptions of §4, equations (13) and (14) remain valid. It is only necessary to sum the power flow of each link (see (21) and (22)),

$$\left.\begin{aligned} \eta_{\mathrm{II\,I}} &= \frac{1}{m_{\mathrm{II}}\,\omega}\sum_{k=1}^{N}\left[g_{\mathrm{II}}^{k}\frac{1}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{I}}^{k})}{|M_{\mathrm{I}}^{k}+M_{\mathrm{II}}^{k}|^2}\right\rangle\right], \\ \eta_{\mathrm{I\,II}} &= \frac{1}{m_{\mathrm{I}}\,\omega}\sum_{k=1}^{N}\left[g_{\mathrm{I}}^{k}\frac{1}{\Delta\omega}\left\langle \frac{\mathrm{Re}\,(M_{\mathrm{II}}^{k})}{|M_{\mathrm{I}}^{k}+M_{\mathrm{II}}^{k}|^2}\right\rangle\right]. \end{aligned}\right\} \tag{24}$$

6. The case of non-rigid coupling

When dealing with industrial links, one has often to consider non-rigid coupling, in this case equations (3) and (4) are no more true and must be replaced by a matrix relation between point velocities after coupling, and coupling forces acting on both systems (F_{I}) and F_{II})

$$\begin{bmatrix} V_{\mathrm{I}} \\ V_{\mathrm{II}} \end{bmatrix} = \begin{bmatrix} A_{\mathrm{I\,I}} & A_{\mathrm{I\,II}} \\ A_{\mathrm{II\,I}} & A_{\mathrm{II\,II}} \end{bmatrix} \begin{bmatrix} F_{\mathrm{I}} \\ F_{\mathrm{II}} \end{bmatrix}. \tag{25}$$

The terms of the matrix characterize mobilities of the link; $A_{\mathrm{I\,I}}$ (resp. $A_{\mathrm{II\,II}}$) is the direct mobility at coupling point with system I (resp. II), $A_{\mathrm{I\,II}}$ and $A_{\mathrm{II\,I}}$ are transfer mobilities. The coupling forces can be calculated equating the velocity after coupling of the link and of the structure, one obtains the coupling forces from decoupled velocities of structures at coupling points:

$$\begin{bmatrix} F_{\mathrm{I}} \\ F_{\mathrm{II}} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} A_{\mathrm{II\,II}} - M_{\mathrm{II}} & -A_{\mathrm{I\,II}} \\ -A_{\mathrm{II\,I}} & A_{\mathrm{I\,I}} - M_{\mathrm{I}} \end{bmatrix} \begin{bmatrix} \tilde{V}_{\mathrm{I}} \\ \tilde{V}_{\mathrm{II}} \end{bmatrix}, \tag{26}$$

where $\Delta = (A_{\mathrm{I\,I}} - M_{\mathrm{I}})(A_{\mathrm{II\,II}} - M_{\mathrm{II}}) - A_{\mathrm{I\,II}} A_{\mathrm{II\,I}}$. It is now easy to calculate powers injected in each system

$$\Pi^{\mathrm{I}} = \tfrac{1}{2} \mathrm{Re}\,[F_{\mathrm{I}}^{*} V_{\mathrm{I}}] \quad \text{and} \quad \Pi^{\mathrm{II}} = \tfrac{1}{2} \mathrm{Re}\,[F_{\mathrm{II}}^{*} V_{\mathrm{II}}], \tag{27}$$

only, of course, if the coupling is dissipative. The powers Π^{I} and Π^{II} are not opposite, and classical SEA relation is no more applicable, on the contrary for non-dissipative coupling, assuming non-correlated decoupled vibrations and smooth variation with frequency of decoupled quadratic velocities, one obtains instead of (12)

$$\Pi_{\mathrm{I} \to \mathrm{II}} = \tfrac{1}{4} |\tilde{V}_{\mathrm{II}}|^2 \left\langle \frac{|A_{\mathrm{I\,II}}|^2 - \mathrm{Re}\,[(\Delta + M_{\mathrm{II}} A_{\mathrm{I\,I}} - M_{\mathrm{I}} M_{\mathrm{II}})(A_{\mathrm{I\,I}}^{*} - M_{\mathrm{I}}^{*})]}{|\Delta|^2} \right\rangle$$
$$- \tfrac{1}{4} |\tilde{V}_{\mathrm{I}}|^2 \left\langle \frac{|A_{\mathrm{II\,I}}|^2 - \mathrm{Re}\,[(\Delta + M_{\mathrm{I}} A_{\mathrm{II\,II}} - M_{\mathrm{I}} M_{\mathrm{II}})(A_{\mathrm{II\,II}}^{*} - M_{\mathrm{II}}^{*})]}{|\Delta|^2} \right\rangle. \tag{28}$$

The coupling loss factors can then be calculated as previously, one obtains for non-homogeneous structures

$$\eta_{\mathrm{II\,I}} = \frac{1}{m_{\mathrm{II}}\,\omega} \frac{1}{\Delta\omega} g_{\mathrm{II}} \left\langle \frac{|A_{\mathrm{I\,II}}|^2 - \mathrm{Re}\,[(\Delta + M_{\mathrm{II}} A_{\mathrm{I\,I}} - M_{\mathrm{I}} M_{\mathrm{II}})(A_{\mathrm{I\,I}}^{*} - M_{\mathrm{I}}^{*})]}{2|\Delta|^2} \right\rangle, \tag{29}$$

$$\eta_{\mathrm{I\,II}} = \frac{1}{m_{\mathrm{I}}\,\omega} \frac{1}{\Delta\omega} g_{\mathrm{I}} \left\langle \frac{|A_{\mathrm{II\,I}}|^2 - \mathrm{Re}\,[(\Delta + M_{\mathrm{I}} A_{\mathrm{II\,II}} - M_{\mathrm{I}} M_{\mathrm{II}})(A_{\mathrm{II\,II}}^{*} - M_{\mathrm{II}}^{*})]}{2|\Delta|^2} \right\rangle. \tag{30}$$

7. Experimental validation

The different expressions of the coupling loss factors previously presented are established with several assumptions and simplifications. Their validity must then be demonstrated on experimental ground, several experiments were done on coupled plates to calculate coupling loss factors from measured mobilities and then to compare energy of plates using SEA relations and direct calculations.

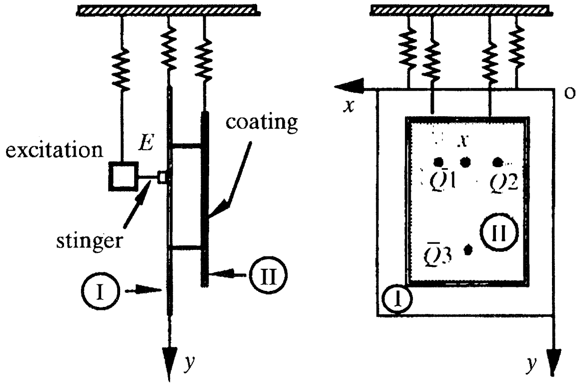

Figure 2. Experiment.

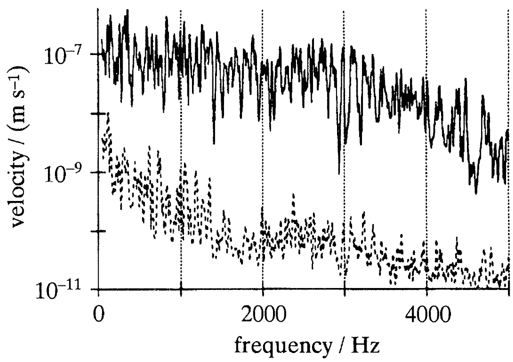

Figure 3. Plates velocities (m s^{-1}) against frequency, without mechanical links. ——, Point X_I on excited plate; ······, point X_{II} on receiving plate.

(*a*) *The case of homogeneous plates*

Two plates were coupled with three mechanical links to connect rigidly transverse displacement but not flexural rotation, this was achieved using stingers (see figure 2). To permit power flow, plate II was covered with a layer of viscoelastic material. Plates were made of steel, they had free edges and were attached by soft springs on a rigid frame. Plate I was 0.7 m × 1.0 m × 0.002 m and plate II was 0.5 m × 0.75 m × 0.002 m. The air gap between plates was 0.085 m. The coordinates of the coupling points $\bar{Q}_i$ and driving point E were given on the general coordinate system X, Y: E(0.22 m, 0.36 m), $\bar{Q}_1$(0.5 m, 0.23 m), $\bar{Q}_2$(0.195 m, 0.23 m), $\bar{Q}_3$(0.35 m, 0.77 m). The masses were 10.35 kg for plate I, 6.40 kg for plate II, 0.0051 kg for each link, 0.003 kg for the dynamic added mass by the force transducer and its screw of fixation. The first resonance frequency of links considered simply supported at each end was 4150 Hz.

Figure 3 shows an experimental result on the plates velocities, when the mechanical links are removed to determine if acoustic transmission is negligible compared with mechanical transmission. The direct excitation is applied at the point E on plate I, and the plate II is excited by the acoustical transmission through the air gap. The ratio of the speeds measured at points X_I (resp. X_{II}) located on the middle of plate I (resp. II) is greater than 100 instead of 2 with mechanical links, so the acoustical transmission through the air gap between the plates may be neglected.

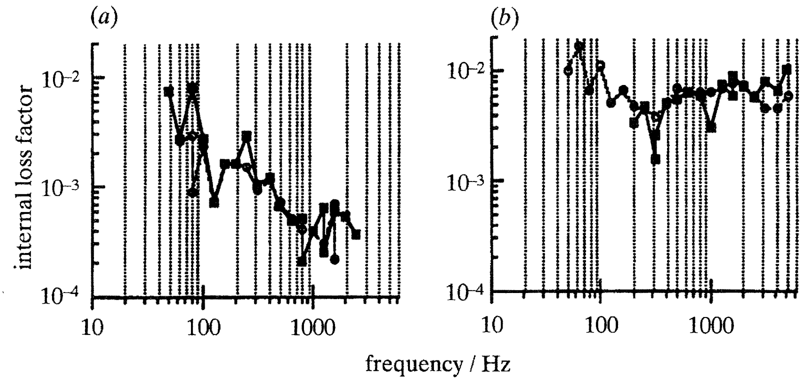

Figure 4. Internal loss factor against frequency in Hertz. (*a*) Plate I, (*b*) plate II. —■—, pink noise excitation; —□—, hammer shock excitation.

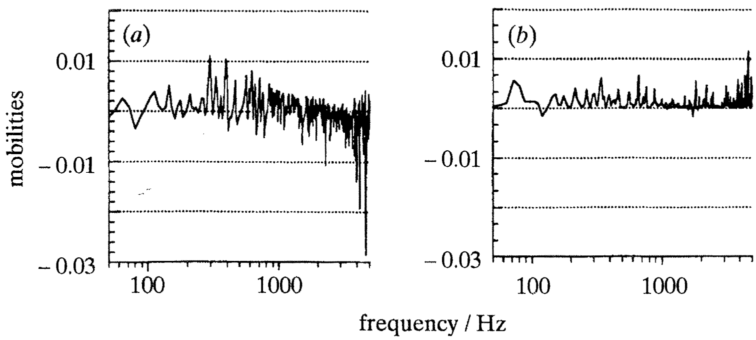

Figure 5. Point mobilities on the receiving plate, at coupling point Q3 against frequency in Hertz. (*a*) Imaginary part, (*b*) real part.

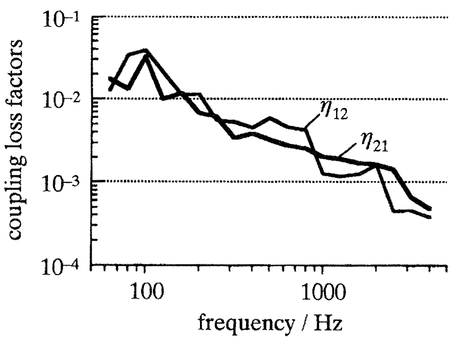

Figure 6. Calculated coupling loss factors expressions (20) and (21) against frequency in $\frac{1}{3}$ octave bandwidth.

Measurements of internal loss factors against frequency of decoupled plates are presented in figure 4. They were deduced from a third octave measurement of the reberation time.

The mobilities of decoupled plates at the junction points were measured, figure 5 shows a typical result. The values of the mobilities do not vary strongly with point of measurement, this is consistent with the homogeneity of the plate. Applying expressions (20) and (21) of the coupling loss factors one get the results shown in figure 6.

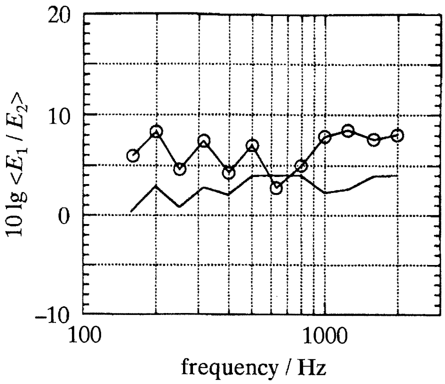

Figure 7. $10\lg(\langle E_1\rangle/\langle E_2\rangle)$ against frequency in Hertz. ——, SEA calculation; --○--, direct measurement.

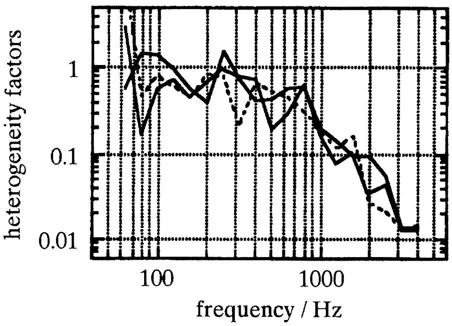

Figure 8. Heterogeneity factors against frequency in Hertz. ——, $\bar{Q}_1$; ------, point $\bar{Q}_2$; ----, point $\bar{Q}_3$.

The two CLF follow the same tendency, they decrease with frequency from 0.01 to 0.001, and remain close together in the whole frequency range.

To verify the validity of calculated coupling loss factors an experiment was done on the two coupled plates described previously. Figure 7 compares the measured plate energy ratio $\langle E_1\rangle/\langle E_2\rangle$ and the SEA prediction obtained, introducing loss factors presented in figure 6, in equation (31):

$$\langle E_1\rangle/\langle E_2\rangle = (\eta_{21}+\eta_2)/\eta_{12}. \tag{31}$$

The SEA prediction overestimates the transmission in general, but the difference between the two curves is reasonable in average. One also notices the variation of the damping loss factor of the receiving plate, in equation (31), introduces strong variation of the SEA energy ratio that can explain the overestimation.

(*b*) *The case of non-homogeneous plate*

To check the validity of expressions (24) of coupling loss factors for non-homogeneous structures, the experiment described in §7*a* was modified. Three added masses (0.175 kg each) were placed on the excited plate at points of coupling.

Figure 8 presents the measured heterogeneity factors associated to each added masses. The three curves are close together and decrease with frequency. This tendency is consistent with the added mass behaviour that blocks the movement when the frequency increases.

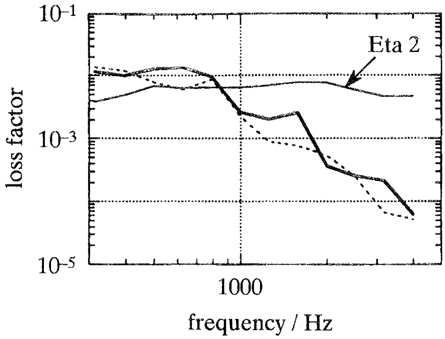

Figure 9. Internal loss factor of structure 2 and SEA coupling loss factors η_{12} (----) and η_{21} (——) against frequency.

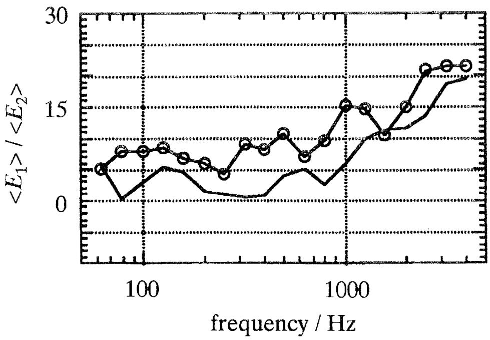

Figure 10. Ratio $\langle E_1\rangle/\langle E_2\rangle$ in dB against frequency in Hertz, where $\mathrm{dB} = 10\log(\langle E_1\rangle/\langle E_2\rangle)$. --○--, Direct measurement; ——, SEA prediction.

The coupling loss factors were calculated according to (24) and the results are presented in figure 9. The coupling loss factors have close values and decrease strongly with frequency (compared with the experiment on homogeneous plates, figure 6). In the high-frequency range, the internal damping of the receiver structure is greater than coupling loss factors and thus governs the transmission. The ratio of the plate energies is close to 2 below 1000 Hz, and increases after, up to 130 at 4000 Hz (see figure 10). The predicted value using SEA coupling loss factors overestimate the transmission as for homogeneous plates, but give the right tendency versus frequency.

8. Conclusion

Determination of SEA coupling loss factors from measured mobilities on decoupled structures have been presented. In the case of two plates coupled in three points, the validity of the expression derived for coupling loss factors, has been demonstrated from an experiment. The heterogeneity factor must be introduced for heterogeneous structure, and used as a weighting function in CLF expression.

The derivation of SEA relations from mobility concepts allows us to introduce non-rigid coupling characterized by input and transfer mobility. The validity of the approach will be checked on experimental ground in the future.

The results given here are applicable for weakly coupled subsystems with high modal overlap. More complex relations would be needed in other circumstances.

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